X GEOMETRICAL OLYMPIAD IN HONOUR OF I. F. SHARYGIN

Final round. Ratmino, 2014, July 31 and August 1

8 grade. First day

8.1. (J. Zajtseva, D. Shvetsov) The incircle of a right-angled triangle ABC touches its catheti AC and BC at points B_1 and A_1, the hypotenuse touches the incircle at point C_1. Lines C_1A_1 and C_1B_1 meet CA and CB respectively at points B_0 and A_0. Prove that $AB_0 = BA_0$.

8.2. (B. Frenkin) Let AH_a and BH_b be the altitudes, AL_a and BL_b be the bisectors of a triangle ABC. It is known that $H_aH_b \parallel L_aL_b$. Is the equality $AC = BC$ correct?

8.3. (A. Blinkov) Points M and N are the midpoints of sides AC and BC of a triangle ABC. Angle MAN is equal to 15°, and angle BAN is equal to 45°. Find angle ABM.

8.4. (T. Kazitsyna) Tanya cut out a triangle from the checkered paper as shown in the picture. Later the lines of the grid faded. Can Tanya restore them without any instruments only folding the triangle (she remembered the triangle sidelengths)?

8 grade. Second day

8.5. (A. Shapovalov) A triangle with angles equal to 30, 70 and 80 degrees is given. Cut it into two triangles in such a way that the bisector of one of them and the median of the second one from the endpoints of the cutting segment are parallel (it is sufficient to find one solution).

8.6. (V. Yasinsky) Two circles k_1 and k_2 with centers O_1 and O_2 touche externally at point O. Points X and Y on k_1 and k_2 respectively are such that rays O_1X and O_2Y are codirectional. Prove that two tangents from X to k_2 and two tangents from Y to k_1 touche the same circle passing through O.

8.7. (Folklor) Two points on a circle are joined by a broken line shorter than the diameter of the circle. Prove that there exists a diameter which does not intersect this broken line.

8.8. (Tran Quang Hung) Let M be the midpoint of the chord AB of a circle (O). Suppose that K is the reflection of M about the center of the circle, and P is a variable point on the circumference of the circle. Let Q be the intersection of the perpendicular of AB through A and the perpendicular of PK through P. Given that H is the projection of P onto AB, prove that QB bisects PH

9 grade. First day

9.1. (V. Yasinsky) Let $ABCD$ be a cyclic quadrilateral. Prove that $AC > BD$ if and only if $(AD - BC)(AB - CD) > 0$.

9.2. (F. Nilov) In the quadrilateral $ABCD$ angles A and C are right. two circles with diameters AB and CD meet at points X and Y. Prove that line XY passes through the midpoint of AC.

9.3. (E. Diomidov) An acute angle A and a point E inside it are given. Construct such points B, C on the sides of the angle that E be the nine points center of triangle ABC.

9.4. (Mahdi Etesami Fard) Let H be the orthocenter of a triangle ABC. If H lies on incircle of ABC , prove that three circles with centers A, B, C and radii AH, BH, CH have a common tangent.

9 grade. Second day

9.5. (D. Shvetsov) In a triangle ABC $\angle B = 60^\circ$, O is the circumcenter, BL is the bisector. The circumcirle of triangle BOL meets the circumcircle of ABC at point D. Prove that $BD \perp AC$.

9.6. (A. Polyansky) Let I be the incenter of a triangle ABC, M, N be the midpoints of arcs ABC and BAC of its circumcircle. Prove that points M, I, N are collinear if and only if $AC + BC = 3AB$.

9.7. (N. Beluhov) Nine circles are drawn around an arbitrary triangle as in the figure. All circles tangent to the same side of the triangle have equal radii. Three lines are drawn, each one connecting one of the triangle's vertices to the center of one of the circles touching the opposite side, as in the figure. Show that the three lines are concurrent.

9.8. (N. Beluhov, S. Gerdgikov) A convex polygon P lies on a flat wooden table. You are allowed to drive some nails into the table. The nails must not go through P, but they may touch its boundary. We say that a set of nails blocks P if the nails make it impossible to move P without lifting it off the table. What is the minimum number of nails that suffices to block any convex polygon P?

10 grade. First day

10.1. (I. Bogdanov, B. Frenkin) The vertices and the circumcenter of an isosceles triangle lie on four different sides of a square. Find the angles of this triangle.

10.2. (A. Zertsalov, D. Skrobot) A circle, its chord AB and the midpoint W of the minor arc AB are given. Take an arbitrary point C on the major arc AB. The tangent to the circle at C meets the tangents at A and B at points X and Y respectively. Lines WX and WY meet AB at points N and M. Prove that the length of segment NM doesn't depend on point C.

10.3. (A. Blinkov) Do there exist convex polyhedra with an arbitrary number of diagonals (a diagonal joins two vertices of a polyhedron and doesn't lie on its surface)?

10.4. (A. Garkavyj, A. Sokolov) A triangle ABC and a point D are given. The circle with center D, passing through A, meets AB and AC at points A_b and A_c respectively. Points B_a, B_c, C_a and C_b are defined similarly. How many does there exist such points D, that points A_b, A_c, B_a, B_c, C_a and C_b are concyclic?

10 grade. Second day

10.5. (A. Zaslavsky) An altitude from one vertex of a triangle, a bisector from the second one and a median from the remaining vertex were drawn, the common points of these three lines were marked, and after this all except for three marked points was erased. Restore the triangle

10.6. (E. H. Garsia) The incircle of a triangle ABC touches AB at point C'. The circle with diameter BC' meets the incircle and the bisector of angle B at points A_1 and A_2 respectively. The circle with diameter AC' meets the incircle and the bisector of angle A at points B_1 and B_2 respectively. Prove that lines AB, A_1B_1, A_2B_2 concur.

10.7. (S. Shosman, O. Ogievetsky) Prove that the smallest angle between the faces of an arbitrary tetrahedron is not greater than the angle between the faces of a regular tetrahedron.

10.8. (N. Beluhov) Given is a cyclic quadrilateral $ABCD$. The point L_a lies in the interior of $\triangle BCD$ and is such that its distances to the sides of this triangle are proportional to the corresponding sides. The points L_b, L_c, and L_d are defined analogously. Show that $L_aL_bL_cL_d$ is cyclic if and only if $ABCD$ is an isosceles trapezoid.