

**X GEOMETRICAL OLYMPIAD IN HONOUR OF  
I. F. SHARYGIN**

**Final round. Ratmino, 2014, July 31 and August 1**

**8 grade. First day**

**8.1.** (J. Zajtseva, D. Shvetsov) The incircle of a right-angled triangle  $ABC$  touches its catheti  $AC$  and  $BC$  at points  $B_1$  and  $A_1$ , the hypotenuse touches the incircle at point  $C_1$ . Lines  $C_1A_1$  and  $C_1B_1$  meet  $CA$  and  $CB$  respectively at points  $B_0$  and  $A_0$ . Prove that  $AB_0 = BA_0$ .

**8.2.** (B. Frenkin) Let  $AH_a$  and  $BH_b$  be the altitudes,  $AL_a$  and  $BL_b$  be the bisectors of a triangle  $ABC$ . It is known that  $H_aH_b \parallel L_aL_b$ . Is the equality  $AC = BC$  correct?

**8.3.** (A. Blinkov) Points  $M$  and  $N$  are the midpoints of sides  $AC$  and  $BC$  of a triangle  $ABC$ . Angle  $MAN$  is equal to  $15^\circ$ , and angle  $BAN$  is equal to  $45^\circ$ . Find angle  $ABM$ .

**8.4.** (T. Kazitsyna) Tanya cut out a triangle from the checkered paper as shown in the picture. Later the lines of the grid faded. Can Tanya restore them without any instruments only folding the triangle (she remembered the triangle sidelengths)?

**8 grade. Second day**

**8.5.** (A. Shapovalov) A triangle with angles equal to 30, 70 and 80 degrees is given. Cut it into two triangles in such a way that the bisector of one of them and the median of the second one from the endpoints of the cutting segment are parallel (it is sufficient to find one solution).

**8.6.** (V. Yasinsky) Two circles  $k_1$  and  $k_2$  with centers  $O_1$  and  $O_2$  touche externally at point  $O$ . Points  $X$  and  $Y$  on  $k_1$  and  $k_2$  respectively are such that rays  $O_1X$  and  $O_2Y$  are codirectional. Prove that two tangents from  $X$  to  $k_2$  and two tangents from  $Y$  to  $k_1$  touche the same circle passing through  $O$ .

**8.7.** (Folklor) Two points on a circle are joined by a broken line shorter than the diameter of the circle. Prove that there exists a diameter which does not intersect this broken line.

**8.8.** (Tran Quang Hung) Let  $M$  be the midpoint of the chord  $AB$  of a circle ( $O$ ). Suppose that  $K$  is the reflection of  $M$  about the center of the circle, and  $P$  is a variable point on the circumference of the circle. Let  $Q$  be the intersection of the perpendicular of  $AB$  through  $A$  and the perpendicular of  $PK$  through  $P$ . Given that  $H$  is the projection of  $P$  onto  $AB$ , prove that  $QB$  bisects  $PH$

**9 grade. First day**

**9.1.** (V. Yasinsky) Let  $ABCD$  be a cyclic quadrilateral. Prove that  $AC > BD$  if and only if  $(AD - BC)(AB - CD) > 0$ .

**9.2.** (F. Nilov) In the quadrilateral  $ABCD$  angles  $A$  and  $C$  are right. two circles with diameters  $AB$  and  $CD$  meet at points  $X$  and  $Y$ . Prove that line  $XY$  passes through the midpoint of  $AC$ .

**9.3.** (E. Diomidov) An acute angle  $A$  and a point  $E$  inside it are given. Construct such points  $B, C$  on the sides of the angle that  $E$  be the nine points center of triangle  $ABC$ .

**9.4.** (Mahdi Etesami Fard) Let  $H$  be the orthocenter of a triangle  $ABC$ . If  $H$  lies on incircle of  $ABC$ , prove that three circles with centers  $A, B, C$  and radii  $AH, BH, CH$  have a common tangent.

**9 grade. Second day**

**9.5.** (D. Shvetsov) In a triangle  $ABC$   $\angle B = 60^\circ$ ,  $O$  is the circumcenter,  $BL$  is the bisector. The circumcircle of triangle  $BOL$  meets the circumcircle of  $ABC$  at point  $D$ . Prove that  $BD \perp AC$ .

**9.6.** (A. Polyansky) Let  $I$  be the incenter of a triangle  $ABC$ ,  $M, N$  be the midpoints of arcs  $ABC$  and  $BAC$  of its circumcircle. Prove that points  $M, I, N$  are collinear if and only if  $AC + BC = 3AB$ .

**9.7.** (N. Beluhov) Nine circles are drawn around an arbitrary triangle as in the figure. All circles tangent to the same side of the triangle have equal radii. Three lines are drawn, each one connecting one of the triangle's vertices to the center of one of the circles touching the opposite side, as in the figure. Show that the three lines are concurrent.

**9.8.** (N. Beluhov, S. Gerdgikov) A convex polygon  $P$  lies on a flat wooden table. You are allowed to drive some nails into the table. The nails must not go through  $P$ , but they may touch its boundary. We say that a set of nails blocks  $P$  if the nails make it impossible to move  $P$  without lifting it off the table. What is the minimum number of nails that suffices to block any convex polygon  $P$ ?

**10 grade. First day**

**10.1.** (I. Bogdanov, B. Frenkin) The vertices and the circumcenter of an isosceles triangle lie on four different sides of a square. Find the angles of this triangle.

**10.2.** (A. Zertsalov, D. Skrobot) A circle, its chord  $AB$  and the midpoint  $W$  of the minor arc  $AB$  are given. Take an arbitrary point  $C$  on the major arc  $AB$ . The tangent to the circle at  $C$  meets the tangents at  $A$  and  $B$  at points  $X$  and  $Y$  respectively. Lines  $WX$  and  $WY$  meet  $AB$  at points  $N$  and  $M$ . Prove that the length of segment  $NM$  doesn't depend on point  $C$ .

**10.3.** (A. Blinkov) Do there exist convex polyhedra with an arbitrary number of diagonals (a diagonal joins two vertices of a polyhedron and doesn't lie on its surface)?

**10.4.** (A. Garkavyj, A. Sokolov) A triangle  $ABC$  and a point  $D$  are given. The circle with center  $D$ , passing through  $A$ , meets  $AB$  and  $AC$  at points  $A_b$  and  $A_c$  respectively. Points  $B_a, B_c, C_a$  and  $C_b$  are defined similarly. How many does there exist such points  $D$ , that points  $A_b, A_c, B_a, B_c, C_a$  and  $C_b$  are concyclic?

### 10 grade. Second day

**10.5.** (A. Zaslavsky) An altitude from one vertex of a triangle, a bisector from the second one and a median from the remaining vertex were drawn, the common points of these three lines were marked, and after this all except for three marked points was erased. Restore the triangle

**10.6.** (E. H. Garsia) The incircle of a triangle  $ABC$  touches  $AB$  at point  $C'$ . The circle with diameter  $BC'$  meets the incircle and the bisector of angle  $B$  at points  $A_1$  and  $A_2$  respectively. The circle with diameter  $AC'$  meets the incircle and the bisector of angle  $A$  at points  $B_1$  and  $B_2$  respectively. Prove that lines  $AB, A_1B_1, A_2B_2$  concur.

**10.7.** (S. Shosman, O. Ogievetsky) Prove that the smallest angle between the faces of an arbitrary tetrahedron is not greater than the angle between the faces of a regular tetrahedron.

**10.8.** (N. Beluhov) Given is a cyclic quadrilateral  $ABCD$ . The point  $L_a$  lies in the interior of  $\triangle BCD$  and is such that its distances to the sides of this triangle are proportional to the corresponding sides. The points  $L_b, L_c,$  and  $L_d$  are defined analogously. Show that  $L_aL_bL_cL_d$  is cyclic if and only if  $ABCD$  is an isosceles trapezoid.