

## PROBLEM SECTION

In this section we suggest to solve and discuss problems provided by readers of the journal. The authors of the problems do not have purely geometric proofs. We hope that interesting proofs will be found by readers and will be published. Please send us your solutions by email: [editor@jcgeometry.org](mailto:editor@jcgeometry.org), as well as interesting “unsolved” problems for publishing in this Problems Section.

*Y. Diomidov, V. Kalashnykov, Trilinear polar and Poncelet’s rotation*

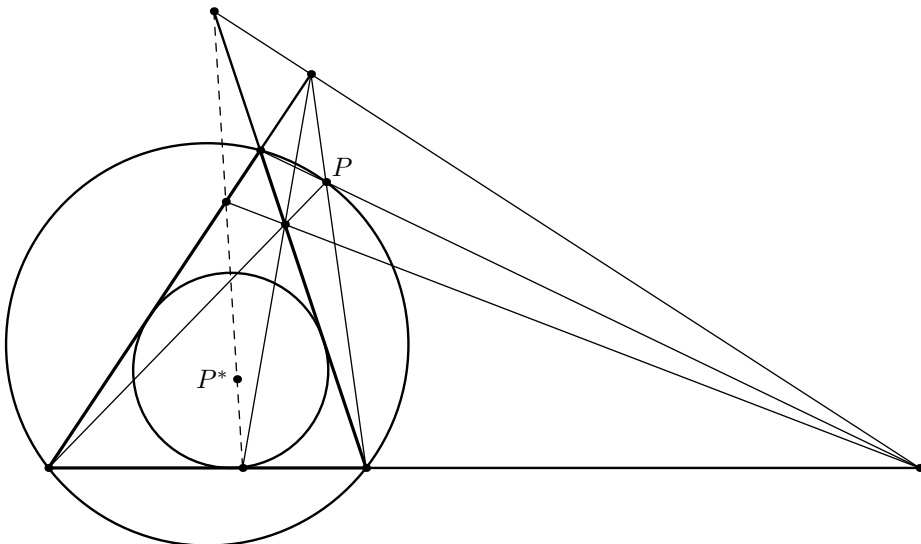
For formulation of the statement of this problem let us recall one corollary of Poncelet’s theorem:

*Let  $\omega$  and  $\Omega$  be inscribed and circumscribed circles of a triangle. Then for any point  $A$  on  $\Omega$  there exists a triangle  $T$  with vertex at  $A$  inscribed in  $\Omega$  and circumscribed around  $\omega$ .*

The rotation of the triangle  $T$  with the point  $A$  we call *Poncelet’s rotation*.

For the following problem the authors do not have a geometrical proof.

**Open Problem.** *Let  $T$  be a Poncelet triangle rotated between two circles and  $P$  be an fixed point on its circumcircle. Then the trilinear polar of  $P$  with respect  $T$  passes through a fixed point.*

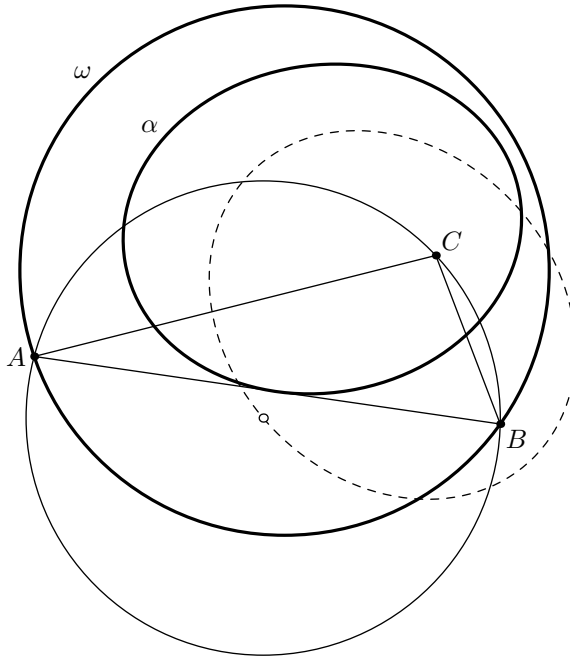


*P. A. Kozhevnikov, A. A. Zaslavsky, A conic of circumcenters*

**Open Problem.** Let  $\omega$  be a circle, and let  $\alpha$  be an ellipse lying inside it. Let  $C$  be a fixed point in the plane. Describe the locus of circumcenters of triangles  $ABC$ , where  $AB$  is a chord of the circle touching the ellipse.

Using elementary algebra approach it is not hard to show that this locus is a conic. Indeed, the locus is a polynomial curve and it is not hard to see that only two points of it lie on the infinite line.

However the authors can not describe the conic geometrically.



*P. Dolgirev, Interesting circle*

**Open Problem.** Let  $\triangle A'B'C'$  be the Gergonne triangle of a triangle  $ABC$ . Draw tangent lines from the vertices  $A, B, C$  to the incircle of the triangle  $A'B'C'$  and denote by  $A_1, A_2, B_1, B_2, C_1, C_2$  intersections of these tangent lines with sides of triangle  $ABC$ . Prove that the six points  $A_1, A_2, B_1, B_2, C_1, C_2$  lie on a same circle.

One of the ways to solve the problem is to show that this circle belongs to the pencil generated by incircles of triangles  $ABC$  and  $A'B'C'$ . For that it is enough to show that for each of these six points the ratio of lengths of tangents to these incircles is constant. But we do not know a synthetic arguments showing that.

Warning: the intersection of cevians  $AA_1$  and  $BB_1$  (and other similar intersection) does not lie on the incircle of  $\triangle ABC$ .

