PROBLEM SECTION

In this section we suggest to solve and discuss problems provided by readers of the journal. The authors of the problems do not have purely geometric proofs. We hope that interesting proofs will be found by readers and will be published. Please send us solutions by email: `editor@jcgeometry.org`, as well as interesting "unsolved" problems for publishing in this Problems Section.

Alexey Zaslavsky, **Brocard's points in quadraleteral.**

Given convex quadrilateral $ABCD$. It is easy to prove that there exists a unique point P such that $\angle PAB = \angle PBC = \angle PCD$. We will call this point *Brocard point* ($Br(ABCD)$) and the respective angle *Brocard angle* ($\phi(ABCD)$) of broken line $ABCD$. Note some properties of Brocard's points and angles:

– $\phi(ABCD) = \phi(DCBA)$ iff $ABCD$ is cyclic;

– if $ABCD$ is harmonic then $\phi(ABCD) = \phi(BCDA)$. Thus there exist two points P, Q such that $\angle PAB = \angle PBC = \angle PCD = \angle PDA = \angle QBA = \angle QCB = \angle QDC = \angle QAD$. These points lye on the circle with diameter OL where O is the circumcenter of $ABCD$, L is the common point of its diagonals and $\angle POL = \angle QOL = \phi(ABCD)$.

Now the problem.

Open Problem. *Let $ABCD$ be a cyclic quadrilateral, $P_1 = Br(ABCD)$, $P_2 = Br(BCDA)$, $P_3 = Br(CDAB)$, $P_4 = Br(DABC)$, $Q_1 = Br(DCBA)$, $Q_2 = Br(ADCB)$, $Q_3 = Br(BADC)$, $Q_4 = Br(CBAD)$. Then $S_{P_1P_2P_3P_4} = S_{Q_1Q_2Q_3Q_4}$.*

This result is obtained by computer and isn't proved.

Lev Emelyanov, **Nagel axis.**

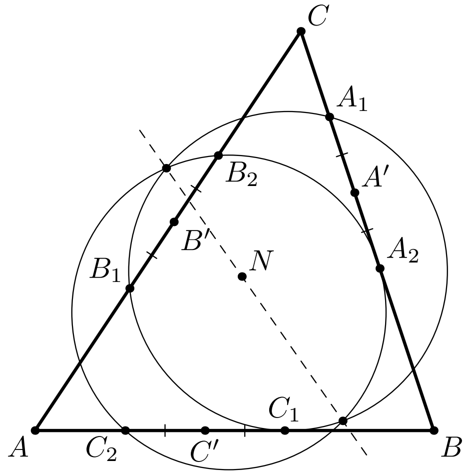
Let N be the Nagel point of a triangle ABC. Let A', B' and C' be the touch points of excircles with the sides BC, CA, AB. Let us consider six points: A_1 and A_2 on the BC, B_1 and B_2 on the CA, C_1 and C_2 on the AB, such that

$$A_1A' = A'A_2 = B_1B' = B'B_2 = C_1C' = C'C_2.$$

Points A_1, B_1, C_1 lie on the rays $A'C$, $B'A$, $C'B$ and A_2, C_2, B_2 lie on the rays $A'B$, $C'A$, $B'C$ respectively.

Then radical axis circumcircles $A_1B_1C_1$ and $A_2B_2C_2$ passes through the Nagel point N, centroid and incenter ABC.

The author does not know the synthetic proof of this fact.



Arseniy Akopyan, **Rotation of isogonal point.**

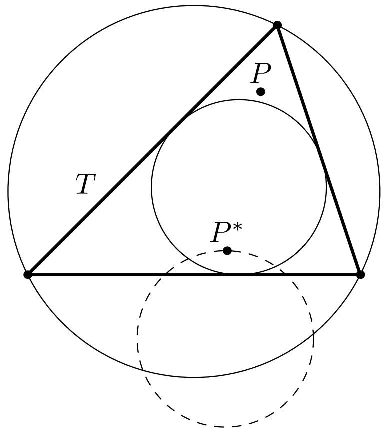
For formulation of the statement of this problem let us recall one corollary of Poncelet's theorem:

Let ω and Ω be inscribed and circumscribed circles of a triangle. Then for any point A on Ω there exists a triangle T with vertex at A inscribed in Ω and circumscribed around ω.

The rotation of the triangle T with the point A we call *Poncelet's rotation.*

The following theorem was found by the author.

Theorem. *Let T be a Poncelet triangle rotated between two circles and P be an any point. Then locus of points P^* isogonal conjugated to P with respect to T is a circle.*



First factful proof was found by François Rideau in [1]. His proof is quite short but not synthetic and use complex number technics. Using his ideas it is not hard to show that the inscribed circle could be substitute by an ellipse.

The synthetic proof of this theorem is not known.

References

[1] F. Rideau. Hyacinthos message 11142, May 29 2005.